

# A Tool for Diagnosing Teachers' Educational Styles in Mathematics: Development, Description and Illustration<sup>1</sup>

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**Abstract:** In his article, M. Hejný (this issue) provided the theoretical background for the development of a diagnostic tool which can be used for the characterisation of a mathematics teacher's educational style. The goal of this study is to describe the tool and show how it can be applied. The tool consists of 20 parameters divided into four areas: (A) beliefs, (B) experience, (C) personality and (D) abilities / competences. The tool has been applied to two short case studies and one longer one. This tool can also inspire a teacher for the improvement of his/her teaching style.

**Keywords:** diagnostic tool, quality of teaching mathematics, a teacher's pedagogical beliefs, a teacher's experiences, a teacher's personality, a teacher's abilities, reflection, transmission and constructivist educational styles, goals of mathematics teaching

## 1 Introduction

This study originated within research led by M. Hejný and stems from his study in this issue (2012). On the basis of his ideas, we have developed a diagnostic tool which can be used to characterize a mathematics teacher's educational style. The tool focuses on the measure to which the teacher develops pupils' creativity in his/her teaching. It enables the teacher to uncover his/her deficiencies in the area of pupils' creativity and points to the possibility of improving his/her work. The researcher can use the tool to analyse teachers' educational styles in detail.

The tool consists of 20 parameters divided into four areas: (A) *Beliefs*, (B) *Experiences*, (C) *Personality*, (D) *Abilities/competences*. The goal of this study is to introduce the methodology of research leading towards the tool, describe the tool in detail and illustrate its application with examples.

When investigating the teacher's educational style, we focused on three target groups: 1. qualified in-service teachers, 2. in-service teachers studying to finish their qualifications, 3. pre-service teachers studying at the author's faculty to become elementary school teachers.

In the last ten years, we have acquired a rich and varied set of research data: video-recordings of lessons, audio-recordings of seminars with teachers, seminar works about the teachers' experience, master theses, written reflections from

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**70** teachers and students on the presented educational situations, written reflections and self-reflections of lessons, pupils' written work, some of them with the teacher's comments, records from observations, etc. Our colleagues also supplied our database with interesting and valuable data.

The rich research material has been analysed from the point of view of pupils' thinking processes and class discussions (Hejný, 2004a, 2011; Jirotková, 2011; Jirotková, Kratochvílová, 2004) and from the point of view of a teacher's educational style (Hejný, 2004b, 2004c, 2004d, 2007, 2008; Kratochvílová, 2004). Various techniques of qualitative analyses have been used (comparative, genetic, classification, etc.), mainly based on techniques of grounded theory (Strauss, Corbinová, 1999).

The research into the teacher's educational style began by the study Hejný (2006) which was inspired by diversity idea of Gray and Tall (Gray at al. 1999): the more creative the pupil, the more varied his/her reactions to challenges. Our development of this idea suggests that: the more creative a teacher's educational style, the more varied the solving strategies of his/her pupils.

## 2 Description of the Tool

Due to the nature of the investigated phenomena, the diagnosis tool which is described here cannot be considered final. The research is on-going and both the set of parameters and their classification into individual areas can be modified with the new data and their analyses. We often faced a problem how to classify a certain phenomenon. For example, the phenomenon of self-assessment is included in the area of Personality, but it can also be put into the area of Beliefs.

Area A – teacher's beliefs – seems to be the key one. It is, however, the result of the teacher's life experience, that is part B. A teacher's personality, area C, is connected to area A in such a way that it is not often possible to include an idea to only one of them. The last area, D, covers the teacher's abilities / competences and is closely connected to the educational processes. This area can be seen as pedagogical content knowledge and content knowledge in the sense of Shulman (1987; see also Janík, 2009). We argue that it is not necessary to define precisely the terms such as belief, life experience, personality, value polarities, etc. as we do not suppose that misunderstanding could arise. As for the key term belief, a concise summary of key research in this area is given in Žalská (2012).

At present, the proposed set of parameters is as follows:

### A. A teacher's beliefs

1. Attitude to mathematics
2. Goals of the teaching of mathematics
3. Educational style (transmission versus constructivist<sup>2</sup>)

<sup>2</sup> In the sense of Noddings (1990).

4. Interactional style towards the pupil, to the class (from attitudinal to dialogical) and possibly their projection into the integration of the pupil with people in his/her environment (classmates, parents, grandparents, etc.)
5. Interactional style towards colleagues, school government, inspection
6. A need to develop his/her competences

**B. Life experience as a springboard for the teacher's pedagogical beliefs**

1. Where it comes from
2. What it concerns
3. What is missing
4. How it is reflected (analysed)
5. Which experience resulted in the shift of the teacher's beliefs

**C. Personality**

1. Self-confidence in the area of pedagogy
2. Self-confidence in the area of didactic
3. Self-confidence in the area of mathematics
4. Self-confidence in the social area (towards colleagues, school management, inspection, parents)
5. Assessment of one's educational style (does it correspond to reality?)

**D. Abilities / competences**

1. Pedagogical: management of class discussion, organisation of work, individualisation, creating and maintaining the climate, work with mistakes, patience, etc.
2. Didactic: pupils' motivation, conception of the ontogeny of concepts, relationships, processes, languages, problem, diagnosis of pupils (understanding their ideas), evaluation of pupils, re-education, etc.
3. Mathematical: knowledge of solving strategies of various types of problems, ability to experiment, to effectively use the trial error method, ability to create generic models both procedural and conceptual, posing problem with required characteristics, etc.
4. Social: interaction with colleagues, school management, inspection, parents.

### 3 Illustration of Parameters

Some of the above parameters will be illustrated by stories of Alice and Matylda.

#### 3.1 Story of Alice

Alice carried out a teaching experiment in grade 3 of an elementary school as a part of the certification process in April 2011. She described and analysed it and in her reflection, she expressed an inner dissatisfaction with the results of her own teaching and initiated discussions with D. Jirotková and M. Hejný. She did not accept their offer that they visit her classes and provide some advice. As an unqualified

72 teacher, she had taught in a transmission way for some years and in the experiment, she was to teach in a constructivist way. She chose the written division algorithm. First, she set the task  $57 : 3 = ?$ . She writes about this part of the experiment:

AL01. "Earlier, I tried to teach children the traditional algorithm of written division. This time, instead of explaining, I had to choose the method of questioning. I asked the children whether they would be able to divide number 57 into 2 numbers divisible by 3. I presumed that they would suggest  $30 + 27$ ."

The teacher expected that the children's answers would be in view with the traditional algorithm. She expected the solution:  $57 : 3 = (30 + 27) : 3 = 10 + 9 = 19$ . However, it did not come. Alice went on:

AL02. "The children suggested pairs  $51 + 6$ ,  $54 + 3$ ,  $48 + 9$ ,  $45 + 12$  (these sums were written in a column below each other)."

Alice seemed to be surprised but she could find the reason for the first division.

AL03. "Some time before that, the children solved  $51 : 3$ ."

Alice could not see a way to proceed from the pupils' suggestions to the required relationship  $57 : 3 = (30 + 27) : 3 = 10 + 9 = 19$ . The pupils took over the initiative and found three more divisions of number 57 into addends divisible by 3. Alice wrote:

AL04. "Radka (a weaker pupil) claimed with enthusiasm: '... there is a lot of solutions, it goes one after another, one column grows, the second decreases by the same number.' Many pupils called out at this moment that it grew and decreased by 3."

Alice acquired new experience with children's activity which she gave space to. In her mind, there was a tension. On the one hand, she was glad that the children were active, on the other hand, she could see that the activity did not lead to the didactic goal of the lesson. She found a compromise:

AL05. "For each division, divide both addends by three and add the results."

The children discovered that number 19 resulted in each case. However, Alice added with disappointment:

AL06. "The number of pupils who understood the reason was small."

At the end of her work, Alice evaluated the whole experiment:

AL07. "I was considering the danger that the children would not be able to solve all the problems. Surely, they do not advance so quickly as with the traditional approach when I was writing five-minute tests with pupils on this topic. They had to complete them within a time limit. These pupils would not mostly be able to do so. On the other hand, with this 'freedom' the pupils demonstrated a certain insight into division and into the connection between arithmetic operations, and great excitement for work."

When speaking to the researchers about her experience from the experiment, Alice mentioned two important facts:

AL08. "I was surprised how much the pupils were able to discover, even the weaker ones. I was pleased that they were active."

AL09. "However, I am afraid that I will not be able to organise such teaching and that the pupils' results in tests will be weak. Moreover, I am afraid of the reaction of some more aggressive parents and sometimes of the school management, too."

### 3.2 Analysis of the story of Alice

The tool of our analysis will be the above set of parameters of the teacher's educational style.

#### *A2.+A3. Goals of the teaching of mathematics and the educational style*

Alice can clearly feel the basic polarity of transmission and constructivist teaching of mathematics (AL01), however, she keeps the goal "pupils count reliably and quickly" and the transmission style (AL07).

#### *A4. Interaction style towards the pupil, to the class*

Alice tries to proceed in a dialogic way, does not criticise the pupils' answers which do not fit her expectations (AL02), evaluates the pupil's autonomy in a positive way (AL04). She leads the pupil in a non-authoritative way towards the chosen educational goal (AL05).

#### *B1. Origin of life experience*

Similarly to most of our teachers, Alice has mostly experienced transmission teaching. As she mentioned several times, she was taught like that when she was a pupil and she has been teaching like that for years (AL01). In the teaching experiment, she acquired one-time positive experience with the constructivist teaching (AL04).

#### *B2. Content of life experience*

In the teaching experiment, the teacher saw that constructivist teaching can bring much more joy to pupils than the transmission one. She realised that pupils were capable of generating much more knowledge than she had thought possible (AL04, AL08).

#### *B3. Missing experience*

During her university studies, Alice met a constructivist educational style in mathematics, she acquired not only theoretical knowledge, but she also saw example lessons taught in a constructivist way. However, only a small part of it was her personal experience.

#### *B4. Reflection of experience*

In the database which concerns Alice, there is no evidence that she tried to rationally reflect on her work. Her reflections are emotional and relate to the failure which she perceives rather fatally.

**74** *B5. Experience causing the shift in the teacher's beliefs*

New experience was not strong enough to having caused a shift in Alice's beliefs in such a way that she would be able to teach in a constructivist way. She was afraid of moving towards the constructivist style of teaching and soon gave up on the effort to change the transmission style.

*C1.+C2.+C4. Self-confidence in the pedagogical, didactic and social areas*

Alice herself introduced two reasons for her resignation: (1) she did not believe that she would be able to manage the situation didactically and (2) she was concerned that most pupils would have problems with subject matter (AL09). From the interview, we can infer the third reason: (3) low self-confidence in the social and partially also in the pedagogic areas.

*C2.+D2. Self-confidence in the didactic area and didactic abilities / competences*

Alice experienced surprise when the pupils proceeded in a different way than expected (AL02). Even though she managed the situation well from the didactic point of view, that is, she found a suitable question which orientated the pupils in the direction of her purposes (AL05), her self-confidence in the didactic area rather suffered, as we noticed several times.

### 3.3 Story of Matylda

Matylda studied the Faculty of Sciences, Charles University in Prague, the teaching of biology and mathematics for pupils at the lower and upper secondary schools. She graduated in 1992 and then she taught at a secondary school for a short time. In 1999, she started teaching at a small rural school. In 2007–2011, she extended her qualifications for teaching at an elementary school. Within her studies, she elaborated a seminar paper and later also graduation work with the topic 'Probabilistic thinking of elementary school pupils'. She was successful with this work at the national level.

The data are taken from the above two works where dozens of copies of pupils' works are included. Moreover, we have transcriptions of many dialogues and notes from correspondence. Matylda writes about her transfer to the elementary school:

MT01. "I consider work in a small rural school to be the most enriching from my past practice. It requires perfect preparation, good planning of the lesson, excellent management of pupils. I have taught from grade 3 to 5, I enjoyed the work, the age of pupils was suitable for me."

Life situation required that Matylda left for a big primary school. She explains:

MT02. "I remained faithful to the same age group (grade 4 and 5), it suits me that the children have not lost their natural desire to discover and learn while at the same time, it is already possible to work on projects with them, discuss things. They orientate themselves in a text, they are more independent."

Matylda did not know how to prepare the teaching experiment. She saw the notion of probability as very demanding for elementary school. Her fears and helplessness intervened in three areas: social, pedagogical, didactic.

- MT03. "Children like to boast at home about what they did at school and thus it is necessary to defend the sense of the experiment before the parents, or ask them for their consent that the child is included in the experiment."
- MT04. "I was afraid that I would not be able to record all the interesting ideas or that some children would not take part in the discussion because they do not find this way of work suitable for them. I hoped that I would be able to evaluate pupils' answers statistically."
- MT05. "I was despondent when introducing the experiment in the class. I thought that I would not be able to explain to children what I want from them. Another important aspect of my fears was that it was an extremely problematic class, heterogeneous as for the social background, nationality and mathematical ability. I had never experienced anything like that."

At the end, Matylda decided to realise the teaching experiment in a written way. She prepared a test/questionnaire for pupils in which she used inappropriately difficult questions and the test was useless for her purposes. Matylda then decided to lead the experiment in a dialogic way. The pupils drew notes from drawing drum, threw dice, discussed things. None of them, however, made any indication of discovering probability notions. Matylda writes:

- MT06. "I was desperate that it was a 'lost' lesson. It was pleasant that the pupils spontaneously said that they liked the lesson. At first I did not understand why but I think I know it now - the children rose above the notions which they did not understand and approached the problem in their way. They experienced action and suspense because they forecasted and drew lots."
- MT07. "I do not think that the notions of probability would be totally inaccessible to children, but it would require that they throw dice repeatedly, drew lots, etc. and only afterwards would think in a 'probabilistic' way based on their experience."
- MT08. "On the basis of the experience from the experiments, I think that it is necessary that elementary pupils really experience the mathematical context in which they work. I recommend a guided discussion so that pupils hear opinions of others and can work with them."

We were interested to what extent the above experience influenced Matylda's educational style after a year. When asked, she said:

- MT09. "For sure, this realisation influenced me together with the way mathematics education lessons [at the university] were taught, when we, students, were in the role of elementary pupils and the teacher was in the role of an elementary teacher."
- MT10. "I teach grade 4 this year. I try to maximally limit communicating ready-made knowledge and algorithms, I have re-evaluated my way of teaching in many ways. I have put children into the role of brilliant, clever, able DISCOVERERS whom I trust and who can manage everything (I praise, I am exhilarated, I act a bit...). I choose a different formulation of questions - 'What do you think...? How is it that...? And what if...?' I am successful very often."

### 3.4 Analysis of the story of Matylda

#### *A1. Attitude to mathematics*

Matylda, contrary to the most of elementary student teachers, has a qualification in mathematics for teaching the lower and upper secondary school. This fact influences her understanding of mathematics as an exact discipline founded on definitions, statements and proofs. Probability is connected to the definition in her mind.

#### *A2.+A3. Goals of teaching mathematics and educational style*

Matylda clearly formulates her effort to teach in a constructivist way (MT10). However, it is not clear to what extent she realises it. When at the end of MT10 she presents questions which she considers to be evidence of her creative teaching, she is limited to the teacher–pupil interaction. There is no mentioning of the discussion within the class.

#### *A4. Interaction style towards the pupil, to the class*

Matylda realises the necessity of mutual dialogue of pupils (MT08), but it is not clear from this attitude to what extent the class discussion is directed by pupils.

#### *A6. A need to improve her competences*

From Matylda's final work, from her big interest in the solution of the didactic problem which she experienced in the first part of her experimental work, it is clear that she looks for ways to solve it both by the study of literature and consultations, and by reflections of her experience.

#### *B1. Origin of life experience*

Matylda's life experience is quite rich and varied. She herself values most the experience which she acquired in a small rural school (MT01) and the experience she acquired during her university studies (MT09). The experiment brings her new experience which she appropriately assesses. (MT08).

#### *B2. Content of life experience*

Matylda's markedly different experiences with mathematics education in two types of university studies are in contradiction. In the area of probability, experience from the study of mathematics and from her teaching at a secondary school dominates. This experience makes it difficult for her to solve the present didactic problem with probability at the elementary school.

#### *B3. Missing experience*

Matylda misses experience with experimenting, mainly with experiments related to the subject matter at the elementary school (MT05, MT06). She has no experience with a differentiated approach to the class. She prepares the experiment in a frontal way, making no allowance for the heterogeneity of the class.

#### *B4. Reflection of experience*

Matylda, to a markedly greater extent than other teachers from our research, believes that she can improve her work by reflecting on experience and its analysis. It can be seen both from her seminar work and from her graduation work (MT08).



*B5. Experience causing the shift in the teacher's beliefs*

The consequence of the preceding reflection is a shift in Matylda's belief that effective teaching lies in widening pupils' experience which brings both knowledge and motivation (MT08, MT10). This experience became an impulse for a series of other shifts which had been going on for more than a year.

*C1. Self-confidence in the pedagogical area*

When leaving the small rural school, Matylda had high self-confidence (MT01). It decreased as a consequence of new and unforeseen experience at the new primary school (MT05). Another important decrease appeared when preparing the experiment (MT04).

*C2. Self-confidence in the didactic area*

Given Matylda's previous didactic success, she had no doubt about her didactic abilities. She attributed problems in the new class to the pedagogical area (MT05). She was taken by surprise by the challenge to carry out an experiment on probability. She could not see how to make this demanding topic available to grade 4 pupils (MT05).

*C3. Self-confidence in the mathematical area*

Matylda was well aware of her knowledge in the area of probability. She had no idea how to present it to grade 4 pupils. She did not know that both in phylogeny and in ontogeny, the notion of probability is a result of the development whose first stage consists of acquiring experience leading to a syncretic pre-concept of probability (chance, possibilities). Matylda saw her initial failure as a challenge, not as a stroke of fate (MT06). Let us add that this is an inspiration for universities preparing teachers. Only rarely do university students learn about the propaedeutic of concepts, relationships, arguments, processes, situations, etc. they are going to teach.

*C4. Self-confidence in the social area*

Matylda fears conflicts with parents which might not want their children to be 'guinea pigs' (MT03).

*C5. Assessment of one's educational style (does it correspond to reality?)*

Matylda is rather self-critical and is surprised by the pupils' positive reaction (MT06).

*D1.+D2.+D3. Pedagogical, didactic and mathematical abilities / competences*

Matylda's key statement in terms of these three parameters is the third and fourth sentence from MT06: "They experienced action and suspense because they forecasted and drew lots." What does Matylda mean by "the children rose above the notions which they did not understand and approached the problem in their way"? We think that when the pupils drew notes with names during the lesson and forecasted what would be drawn, their considerations were totally incomprehensible to her. In Matylda's protocols we can read pupils' statements such as "Jana will be the first to be drawn as she is always the first" or "Eva will be the first because her name has only three letters and such a note is lighter and is lying on the surface". From the

**78** pedagogical point of view, Matyllda could no longer direct the pupils' work. On the other hand, she gave them enough space for their creativity. From the didactic point of view, Matyllda has no other problems prepared to widen the pupils' experience base. From the mathematical point of view, Matyllda is captured in her images of probability as a topic of teaching. She has no knowledge of its propaedeutic.

In this respect, we can refer to results of research of J. Kohnová (1995, s. 44). She presents answers of teachers to the question what is the source of help in their education in the areas of a) content, b) methods of teaching. In both cases, self-study comes first and colleagues second. Universities have a score of 15% in the content area and 13% in the methods area. The research is rather old and we can hope that the numbers might have changed with the change of the university education of teachers. Statement MT09 supports this hope.

## 4 Case Study of Jitka

Contrary to the previous two stories, we have a ten-year database about Jitka which includes her seminar papers, master thesis, rich correspondence between us, with our colleagues and her pupils' parents, many video recordings of her lessons, many interesting mathematical products of herself and her pupils, etc. Moreover, we have been cooperating with Jitka for more than ten years.

### 4.1 University study

Jitka wanted to study at the Faculty of Education but was not accepted on several occasions. She worked in an after school club and in a kindergarten. After nine years since her secondary school leaving examination, she welcomed an opportunity to teach in grade 4. She was confident enough that she would manage the task. She was taken by surprise that she had to repeat her explanations for some pupils and that not all of them were interested in mathematics. She wanted all pupils to be active. She found partial help in scientific literature, nearly none at her colleagues who, on the contrary, toned down her effort to find more efficient teaching methods. Pupils' positive reactions were her best support. She came to the conclusion that the discrepancy between how she was teaching and how she would like to teach can only be solved by her education.

After eleven years after her secondary school leaving examination, Jitka entered the Faculty of Education with clearly formulated needs. However, at the same time, she was afraid whether her mathematical knowledge would be sufficient. With joy, she stated about her university classes: "... no rules were required from me, my life experience and common sense sufficed. Moreover, I kept getting strong arguments against sceptic colleagues at my school. "

Jitka chose to write master thesis in geometry. She felt her own deficiency in this area but she could also see that in geometry, there was enough space for math-

emational and didactic creativity. In the conclusions of her thesis, she writes: "It is strange to consider conclusions at the moment when I have a feeling that many ways are opening to me only now."

## 4.2 Project

A remarkable result of her thesis, which won a nation-wide competition, inspired us to include Jitka in an international project Comenius. She felt strengthened by these successes and soon acquired a new degree (a kind of postgraduate degree, between the graduate master degree and PhD degree).

In the project, Jitka had to find a cooperating colleague in her school. She writes about it: "... my colleagues did not want to join the project, they feared failure and that none would value it anyway, rather on the contrary." At the end, she persuaded one colleague but it was necessary to encourage and motivate her all the time. (Jitka used the experience which she acquired from this cooperation later when she worked as a teacher educator who also had to encourage and motivate.) When we started working on a new mathematics textbook for grade 1, Jitka was very interested in the material which we were preparing and made arrangements that she could teach in grade 1 at her school and pilot the new textbooks.

## 4.3 Interaction style towards colleagues

In the summary reflection in school year 2006/2007, Jitka writes (shortened): "I start in grade 1 with which I have had no experience and I am interested in 'our mathematics'. The teacher responsible for all classes in the same grade is an experienced teacher, Maruska. We have 4 parallel classes. Already in September, a tension appears between Maruska and me. The reason lies in different opinions of the teaching of mathematics and reading. Paradoxically, just with my opinions of the teaching of reading, I won experienced colleagues and thus became the authority in our team. Through the success in teaching reading, my colleagues began to be interested in the teaching of mathematics, too. An important moment came when we were teaching 'crossing the ten'.<sup>3</sup> My colleagues did not believe that it was possible to teach addition without the procedure 'crossing the ten'. Maruska claimed that it was nonsense and that I would burn my fingers. When I have so much experience as she has, she continued, I will find it out.

On the one hand, I believed in what I was doing, on the other hand I had doubts. Nevertheless, also some other colleagues tried to avoid crossing the ten procedure and were excited that the children indeed managed doing it and without any explanation.

<sup>3</sup> In most Czech textbooks, addition of the type  $7 + 5$  is given by the procedure  $7 + (3 + 2) = (7 + 3) + 2 = 10 + 2 = 12$ . This procedure is called 'crossing the ten' and its practising is devoted a lot of time.

This happened at the moment (May) when crossing the ten was in the teaching plan. Maruska had the whole folder of worksheets. She explained to the colleagues that otherwise the children would not understand addition. She decided that I could do it in any way I like but the others would do it properly. Till the end of the school year, the colleagues practised crossing the ten procedure according to these worksheets.”

In the following school year when Jitka started to officially pilot our textbooks, tensions between Maruska and Jitka concerning the teaching of mathematics escalated. Two of her colleagues had doubts. Jitka had to face some parents' dissatisfaction. She writes: “I organise meetings with mathematics for them. I have fears and I hope that it will help. A dissatisfied parent is a warning sign for the head teacher and he could ban my next initiative, but I want to teach in this way. For the first time, I ask my colleagues for comparative tests. Tactically, I do not speak about the results which are markedly better in my class before the management of the school. I want my colleagues to trust me that a possible new comparison of the children will not be unpleasant for them.” Results of Jitka's teaching were best seen in grade 3 in the nation-wide competition Cvrcek (i.e., a modified Kangaroo competition for the 2nd and 3rd grade pupils). In 2009, from more than 8000 pupils of the Middle Bohemia Region, only 25 got the maximum number of points. Five out of these 25 were from Jitka's class. Her weakest pupil's result was above the whole region average.

Jitka's success in her pupils' knowledge and their approach to mathematics had four important consequences: (1) Jitka has become a teacher educator and with great success disseminates ideas of scheme oriented education in the whole Czech Republic. (2) By her responsive approach to her colleagues, within three years, she won all primary classes teachers of her school for the way of teaching she advocates. (3) She has become a co-author of our teaching materials. (4) She started as a mentor of continual teaching practice of the faculty students and as their teacher at the university. Many students highly appreciate what they are learning from Jitka.

#### 4.4 Analysis

Jitka's case enables us to highlight the interconnection of parameters from section 2. We will focus on the parameters which were illustrated in section 3 only scarcely or not at all.

Jitka studied a secondary pedagogical school and thus had a weaker mathematical education. The university studies which emphasised creativity enabled her to advance in mathematics markedly. She repeatedly experienced joy from solving problems and this motivated her in an on-going way to widen and deepen her mathematical knowledge. (A1, A6)

Already when she came to the university, Jitka felt that the main goals of the teaching of mathematics should be in the development of pupils' thinking, not in the pace of their calculations. During her studies, she was acquiring theoretical knowledge which she immediately used in her class, and thus she interiorised it. Her conviction shifted from the intuitive level to the conscious level, which was support-

ed by her experience and by arguments. By the targeted connection of theory and practice, her teaching became research in action. While teaching, she was able to solve didactic problems which appeared during the preparation of the textbook. (A2)

Jitka's beliefs were close to the constructivist educational style already at the beginning, even though she was taught in a transmission way. She could not use it in her teaching completely because there was a lack of suitable teaching materials. When she first met such materials within the international project and then when trialling the textbooks, her teaching became strongly constructivist. (A3)

Already as a secondary student, Jitka acquired first experience in mathematics education – the teacher asked her to explain subject matter to her classmates and Jitka was successful in it. When she started to work at school as an unqualified teacher, she did not let her experienced colleagues guide her towards transmission teaching. Gradually, she widened her didactic experience in the above activities. Her on-going self-reflection contributed to the speed of her development. Parallel to didactic experience, Jitka acquired pedagogical experience, too, namely, she was better able to understand her pupils. All of them, not only the successful ones, liked her and trusted her. (A4, B1, B2, B4, B5) It is understandable that due to this experience Jitka's self-confidence both in the pedagogical and didactic areas grew. (C1, C2) The comparison of parameter B4 at Alice (without self-reflection), Matylda (occasional) and at Jitka (on-going self-reflection) shows that this parameter has a strong information value in terms of the quality of the teacher's teaching style. The same conclusion has been made by V. Spilková (2011, p. 133).

The social context in which Jitka was working was not favourite to her in the first years. She herself persuaded all colleagues, school management and distrustful parents of the rightness of her way which bears witness of the interaction style of an exceptional quality which guided her. (A5, D4) A teacher's social interaction style strongly depends on his/her social self-reflection and that is strengthened by social success. (C4) At present, Jitka is a confident and modest teacher with an on-going need to reflect, evaluate and improve her work. (B4, C5, A6) Her effort to educate pupils, and not only those from her class, is still markedly dominating her action. (A2)

## 5 Conclusions

The theoretical considerations above are closely related to the activity of D. Jirotková and M. Hejný who strive to enhance constructivist teaching in the Czech Republic. In the section Case study of Jitka, it is mentioned that the mathematics textbooks written by them and their co-operators for elementary school pupils lead to very good results in terms of pupils' mathematical knowledge and their attitude towards mathematics, provided rules of scheme-oriented education (see Hejný, 2012, this issue) are followed. Similar results are reached in other classes taught in a similar way. The story of Alice, however, shows that textbooks themselves are not sufficient for creative teaching. For sure, Alice has been striving to teach crea-

82 tively, has been in contact with the theory of scheme-oriented education, has seen lessons taught in this way, but nevertheless, as she writes in AL09, remains faithful to a transmission educational style.

The example of Alice and some other teachers led us to believe that a teacher who teaches in a transmission way has to overcome various obstacles when he/she wants to change his/her educational style. There are two obstacles in the case of Alice: the first is her pedagogical beliefs about the precedence of the pupil's traditional performance (reliability and speed of calculations); the second is a low pedagogical and social self-confidence. Also in other cases which are known to us, the above obstacles play a key role in the failure of an attempt to change an educational style. According to our present experience, we see value polarities as the key factor of the acceptance or refusal of the educational shift. On the one hand, there are pupils and their joy and performance; on the other hand, there is fear of institutional failure and social problems. Alice's statements refer to this:

"I was surprised how much the pupils were able to discover, even the weaker ones. I was pleased that they were active. However, I am afraid that [...] the pupils' results in tests will be weak. Moreover, I am afraid of the reaction of some more aggressive parents and sometimes of the school management, too."

Matylda's case is the opposite one. Matylda definitely places the value of the pupils' development at the highest place and she is strongly emotionally influenced by her results when she used new educational procedures. Her need to see the joy of pupils keeps alive her effort to look for and develop educational tools which cause the joy.

The case of Jitka is exceptional. Since the beginning of her career as a teacher, she has been looking for ways to inspire pupils, to give them more than just formulas and algorithms. Once her need met a solid base in her university studies, she immediately took the opportunity and in several years, she has become not only an excellent teacher but also a person who is able to motivate others by her example.

Dozens of investigated cases (three are given in this article) lead us to the conclusion that the quality of the teacher's educational style in mathematics is given by his/her *need to see the joy of pupils from their intellectual growth*. This thesis is in full agreement with Freundenthal's idea *Mathematics is a human activity*.

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