Dispute in Mathematical Classroom Discourse – "No go" or Chance for Fundamental Learning?

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Abstract: Disputed points in school mathematical discourse are rarely discussed. If mathematics is seen as a "ready-made" discipline which teaches dealing with right and wrong only, mathematical questions which require debating do not play an essential role. Looking at the real teaching practice shows indeed quite a different picture: Mathematical questions which require debate and justifications appear in the course of mathematical interaction in different places and are dealt with in different intensity. In this article we present small group discussions within the project *Probing and Evaluating Focusing Interaction Strategies in Elementary Mathematics Teaching* (*ProFIT*). The discourse is based on disputed points which turned out to be a promising way of supporting mathematical oriented discussions, a deeper understanding of ambiguous learning situations and the chances for fundamental learning.

Keywords: dispute, discursive learning, focusing interaction strategies

1 Introduction

Common reform efforts emphasise mathematics as science of patterns (Devlin, 1994; Wittmann, 1995). To communicate in mathematics a system of symbols is needed. According to Steinbring (2005), it is impossible to communicate about mathematics in a direct manner. Mathematical signs and symbols do not directly refer to real objects, but symbolise the structure, relationship, patterns, arrangements, connections, etc. between different mathematical objects in an operative way which lies in the special nature of mathematical signs and symbols (Steinbring, 1999, p. 13). The dilemma is that mathematical signs and symbols are often used in many different ways and thus gain an ambiguous status. A mathematician has several convictions and deductions in mind for dealing adequately with the defined symbol systems and its derivations in different situations. However, as pupils at elementary school are in a learning process, they begin to construct their own understanding of mathematical signs and symbols and how these are used correctly in different mathematical situations. Certainly – like in other subjects – young pupils do not always use the signs and symbols in the conventionally correct manner from the beginning. A mutual discourse gives them the opportunity to (re-)phrase their interpretations to each other. Of course, their interpretations are sometimes inconsistent or contradictory and lead to dispute or disagreement among the interlocutors.

 Recent research reports identified dialogical learning as essential to the teaching and learning of mathematics (e.g. Sfard, 2000; Steinbring, 2005; Nührenbörger & Steinbring, 2009). However, the teaching discourse cannot ensure fundamental learning on its own. The way of teaching and the special teacher's role in the discourse essentially affect how profound dialogical learning will be for the participants. Through analysing discourse, Wertsch and Toma (1995) distinguish between univocal discourse (one-way communication from a sender to a listener) and dialogical discourse (constructing meaning by joint communication). While many studies focus on the teachers' proficiency in the sense of gathering mathematical know-how and on the educational knowledge of mathematics teachers (e.g. Kunter et al., 2011), further research is needed to understand the role of teachers in supporting the discursive negotiation of mathematical meanings and, by that, supporting the pupils' relational learning by classroom interaction.

2 Perspectives on Mathematics: Mathematics Requires Construction as well as Interpretation

Both at school and in science, mathematics displays a highly complex system of signs and symbols. Mathematicians use these signs and symbols routinely and naturally – usually without thinking explicitly about its "correct" use. This is not meant to point to deficits of young pupils' mathematical activities compared with mathematicians' research work. First of all, this should make explicit the specific epistemological challenges of understanding mathematical knowledge. Duval explains this position as "paradoxical character of mathematical knowledge":

> […] there is an important gap between mathematical knowledge and knowledge in other sciences such as astronomy, physics, biology, or botany. We do not have any perceptive or instrumental access to mathematical objects, even the most elementary, […]. We cannot see them, study them through a microscope or take a picture of them. The only way of gaining access to them is using signs, words or symbols, expressions or drawings. But, at the same time, mathematical objects must not be confused with the used semiotic representations. This conflicting requirement makes the specific core of mathematical knowledge. And it begins early with numbers which do not have to be identified with digits and the used numeral systems (binary, decimal). (Duval, 2000, p. 61)

However, especially these interpretations, which often stay implicit in the discipline of science, demand cognitive processes of mathematical understanding which stay at first concealed for the learning pupils. Therefore, in the context of primary school, teaching cannot be based on ready-made mathematics that is transmitted to the pupils as a recipe for using numbers. For pupils, mathematical signs and symbols become an independent requirement of construction and interpretation.

Interpretations are not steady but vary depending on the situational context and the interpreting person. That means different interpretations can exist at the same time, which have to be weighed against each other. Especially at primary school, this is problematic, as many different visual representations are used and various concepts are formed. Therefore, an *empirical* or a *theoretical* view (Steinbring, 1994) towards the mathematical object can be held. The empirical view is related to the concretely perceived objects and their descriptions. The theoretical view does not see mathematical objects as things but mathematical relations are built through them.

2.1 Features of interaction at school about mathematics

With this perception of mathematical concepts, *interaction* at school *about mathematics* makes a difference. The pupils themselves have to participate actively in the interaction and they have to verbalise their interpretations. That means the teacher has to be open towards the pupils’ personal constructs of interpretation and attempts at reasoning. Wood (1994, 1998) characterises this teaching behaviour as the interaction pattern of *focusing* and distinguishes it from the interaction pattern of *funnelling*. She points out the importance of involving the pupils actively in the interaction process, making the mathematical content as well as the communication itself accessible for them, asking them for mathematical explanations and reasons and sharing their ideas during the discourse. The pupils’ different points of view become visible, because of this requirement for the pupils.

In such an interaction, the teacher does not intervene in a controlling or regulating way and the children will not take over the teacher’s view immediately. The teacher explicitly asks the pupils to explain their own views. By explaining mathematical signs and symbols, the pupils refer to “objects/reference contexts” which are in part very different from each other. So it is important to understand that – in our context – the concepts “sign/symbol” and “object/reference context” do not stand on their own in the discourse. The concepts “sign/symbol” and “object/reference context” are, metaphorically spoken, certain means of discursive negotiation of meaning and they do not directly provide the meaning in question, but they serve the elaboration of the epistemology of mathematical knowledge in the social interaction context (Steinbring, 2005, p. 12ff.).

Of course, pupils at primary school are not always able to express themselves decidedly and explain mathematical patterns and structures precisely. The empirical research of Miller shows that “children, who […] are only 7 to 8 years old and older, know basic cognitive and linguistic-communicative techniques and argumentations and are able to consider their own normative point of view or parameter of value and, at the same time, the one of the opponents in argumentations” (Miller, 2006, p. 47, translated by the authors). Consequently, the statements in mathematical classroom discourse do not underlie this objective indefeasibility which the discipline of science, being formally abstract, allows mathematics. Instead, mathematical objects underlie an ambiguity which provides numerous links for discussions in lessons if the teacher is aware of this ambiguity and introduces it as a constitutive element in his or her classes. Already in 1990, Voigt refers to the tension between the childlike endowing of meaning and the teacher’s didactic-curricular motivated interpretations referring to mathematics. But how is it possible to realise that this endowing of meaning as well as the teacher's interpretations referring to mathematics become subject of the interaction?

2.2 Fundamental learning by disputed points in school mathematical discourse

In this article, we present the possibility of emphasizing the participants' different interpretations in the interaction. Voigt stresses the importance of enduring and eliminating competitive ambiguity in classroom interaction (Voigt, 1990, p. 308). A focused interaction, related to the differences and relations between ambiguous objects and the partly competitive interpretations, can lead to dispute. If the interaction does not cease, it provokes the pupils to discuss the mathematical justified disputed points and may be called "focusing".

It is a challenge of such an interaction to perceive and realise the crucial point: "Collective argumentations require perception of inter-individual coordination processes by the interlocutors and they are an attempt to develop collective solutions […]" (Miller, 1986, p. 24, translated by the authors). If the disputed point is not recognised, it cannot lead to fundamental learning processes. Referring to Miller, "fundamental learning processes" lead to new structural (social-)cognitive problem solving and to a progressive and more adequate recognition of a higher tier concerning the world of nature, the social world and the world of the own inner being (1986, pp. 9–10). *Discursive contexts of discovery about new beliefs and new knowledge* (Miller, 1986, pp. 246–341) are the basis for fundamental learning in collective argumentation.

> A discursive context of discovery can be identified and located as the network, constituted by collective argumentation in which possible thoughts and partial arguments develop, that mediate between thesis and antithesis and possibly establish a consensus between both. […] the corresponding discourse processes are not at all arbitrary. They depend on how the participants proceed for gaining a joint understanding about what the question of debate is in their dispute. (Miller, 2006, pp. 216–217, translated by the authors).

According to Miller, a crucial point "is not usually a generic normative utterance, that means utterances of the type "In general, one should…", but singular normative utterances of the type "In situation *s* person *x* should not have done operation *y* but operation *z*". The normative elements of a daily life context or of a general socio-cultural system of values are normally not controversial issues, but, generally, the application of collective moral codes in concrete cases of conflict is controversial." (Miller, 2006, pp. 45–46, translated by the authors)

However, in mathematics we do not have a moral code, but a sophisticated system of signs and symbols, which are subjected to manifold relations regarding to conventions and deductions. Through this, a reformulation of Miller's proposition may look like this: "The student *x* is more likely to make the interpretation *y* than *z* of mathematical signs and symbols. Meanwhile, he or she should refer to the object *r*.” Without a doubt, this refers to the one mathematically correct answer. But especially utterances constructed in such a manner could be matter in dispute, if, in accordance to Wood, the teacher makes an argumentative discussion possible and the discursive argumentation of different meanings becomes subject of classroom interaction. Whether or not the discourse leads to consensus does not matter for fundamental learning taking place in these situations:

“For the triggering of structural learning processes, getting a consensus about the question of dissent is not even necessary. Only a method of mutual understanding of differences is required; and the more complex and non-transparent the differences are, the more radical and profound learning can be.” (Miller, 2006, pp. 217–218, translated by the authors)

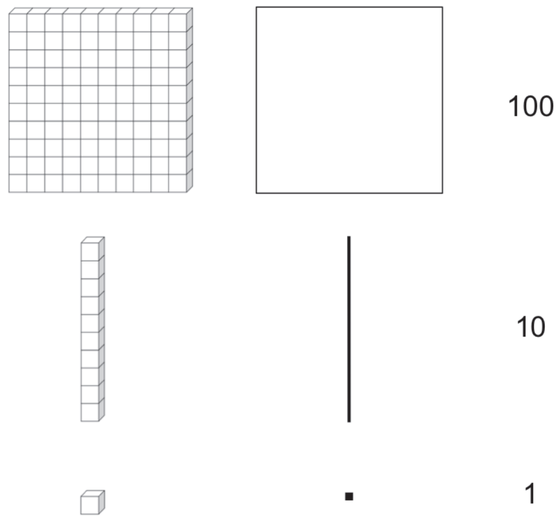

Figure 1 Iconic and symbolic representation of the Dienes-material

For example, in (German) primary schools the use of Dienes-material[1] is common practice. According to Voigt, a didactic-curricular motivated, mathematically related (teacher’s) interpretation would mean that squares represent hundreds, bars represent tens and dots represent ones (Figure 1). However, possible children’s productions of meaning (influenced by their daily life or by [mathematics] lessons) could be – for example – the interpretation of the squares as ones-cubes, the bars as tallies and/or the little dots as reversible tiles. These interpretations might be partly competitive, many more might be possible and – depending on the point of view – they might be matter in dispute.

1 Named after Zoltan Paul Dienes

Another example is the ambiguity of a number-line without numbers (Figure 2). The little boxes can be filled in differently. For example, the starting point, the section, the unit and/or the scale can be varied, especially because these choices could evolve into possible crucial points in the interaction.

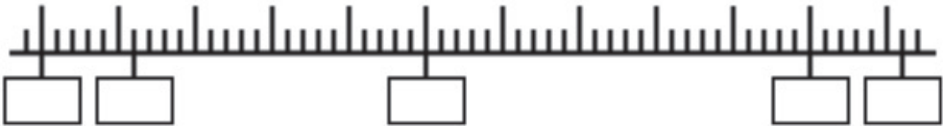

Figure 2 Number-line without numbers

3 Data Collection

The study "Probing and Evaluating Focusing Interaction Strategies in Elementary Mathematics Teaching (ProFIT)" was conducted at a primary school in year 3 and 4 in Germany. Having the theoretical background in mind, the teacher's role in the project "ProFIT" should be to focus the mutual attention at the crucial point of a problem, to raise a question in order to hand the discussion back to the pupils and to give them the responsibility of clarifying the mathematical situation themselves (Wood, 1994). Through such a form of interaction, needs of mathematical reasoning are emphasised as a central research topic and different pupils' interpretations have to be compared. So the main research interest is to develop and reconstruct discursive mathematical interaction as a theoretical construct of mathematics education called "focusing interaction strategies", in which the central characteristics of negotiation processes between teacher and students are classified. Hence, the research interest is directed on two perspectives:

- How can the teacher make a need for more mathematical reasoning more accessible for pupils? And: In which way can the needs be developed in the common discourse?
- How can pupils understand a mathematical reasoning need? And: How are they able to agree on it?

Five teachers participated in the project. In several videotaped small group discussions, each of them discussed four mathematical topics with several groups of four children. The topics' main characteristics were the openness with regard to ambiguous conventional aspects (Steinbring, 1994, see the two examples above) and operational connections in problem-based tasks (Wittmann, 1995). So the crucial points of these tasks can be the different interpretations of the mathematical signs/symbols or the different ways of solving problems. The theoretical construct is particularly elaborated by theory-based, interpretative analysis of the video data of these experimentally planned mathematical discourses with pupils.

4 Data Analyses

For the data analyses we take two theoretical points of view into consideration: the epistemological and the evolving steps of debate in a broader unit of meaning. The epistemological point of view deals with the fact that it is impossible to analyse discourse without seeing the contributions of the interlocutors in the interaction. Hence, the outcome of an interaction often becomes only visible in the particular utterances. Against this background, the data analysis starts with the epistemological analysis of the meanings of mathematical signs and symbols being interactively constructed by the pupils (Steinbring, 2005). This epistemological analysis is embedded in the analysis of a broader episode, considering the three evolving steps of dispute: Initiation – Maintenance – Breakup. Beginning with a potential point of dispute based on different interpretations; whether the interaction about the differing representations is pursued or dropped will be analysed in the next step. In the latter cases, the dispute breaks up at once. If we follow the interaction, we look more closely at the way of maintenance. This is analysed with two questions in mind:

- Is the interaction more teacher-centred, which means that the pupils' changing and partly not conventional interpretations are taken as "mistakes"? In this case, the "correct" conventional use of signs and symbols is transmitted by the teacher and the interaction is led to consensus. Such an interaction is regulated by the teacher's logic of interaction.
- Or does the interaction focus on the special mathematical content? In this case, the question of debate is not only realised, but substantiated and deepened. A possibly emerging consensus is rather based on the subject matter's logic instead of the logic of interaction.

4.1 First Example: Dispute about number representations

The task "Find different representations for the number 417!" promotes classroom discourse about the previously produced solutions of the pupils, as shown in figure 3.

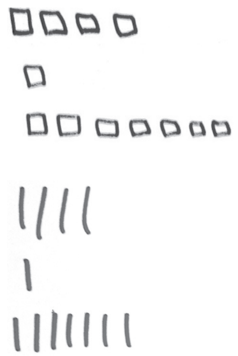

Figure 3 Pupils' solutions for representing 417

Here, important differences to conventional representations become visible. Earlier, there was a conventional use of the Dienes-material: “little cubes” representing ones, “bars of each ten cubes” representing tens and “squares of each a hundred cubes” representing hundreds. In contrast to this interpretation, some of the elements of the Dienes-material now have been given another representing role. In figure 3, two representations of the number 417 are shown, one using the bars and the other the squares of the Dienes-material in a new way. For representing a number value, the perceptible form of squares or bars is not taken, but the value of hundreds, tens and ones depends on spatial relations between the same elements. These two representations are in the focus of discourse in the following episode.

Ben interprets the “squares” representation of 417:

B The one with the squares, there are always, um (…). On top there are four squares, that’s supposed to be the four (…), the four hundred and the one is of the, seventeen the one and those seven squares are supposed to be the seven of seventeen.

T Can you show me that more precisely? I cannot see that. (…)

B Well this (*goes over the four squares in the first line with his finger*) is supposed to be the, four hundred the four squares the single square (*points with his finger at the single square in the middle*) is supposed to be (…) ten and that (*goes over the seven squares at the bottom line with his finger*) are supposed to be seven.

In his explanations, Ben exclusively refers to the representation with “squares”. He attributes a numerical value to every square, varied by different spatial relations within the representation. His choice of words “is supposed to be” emphasises the interpretation character of his utterance. The teacher’s intervention leads him towards pointing precisely at the representations, but not towards verbalising the arrangement any further.

Manuel carries out another interpretation:

M That are, um, but however the bars are the tens and the units (*points at the poster with the bars*).

He takes the representation with the bars into consideration and interprets the bars as “tens and units”. What he understands exactly as ones and tens does not become clear. Possibly, he interprets them as tallies (ones) or as the tens-bars of the Dienes-material. By saying “are” he takes a more or less definitive perspective towards the representation, which indicates that each symbol represents something special, possibly fixed. With the German word “doch” (in English “but however”) Manuel separates his own interpretation from Ben’s, so that his utterance as response to Ben’s has the potential for being in dispute.

Ben negates Manuel’s arguments indirectly:

B That is the same as this one just with bars (*points in turns to the poster with bars, then at the representation with squares*).

What is new in Ben’s contribution? He accepts the dissent and, hence, takes both representations into account. He speaks about “the same”, but he does not show his understanding of “the same”. The representations are not the same, but they have the same structure and the individual symbols have the same local positions. This defends Ben’s argument that each of these presentations could be 417.
Frank joins in:

F Um, the ones are the, these little cubes. And the big, and the big squares are the hundreds and the bars are the tens.

Frank’s utterance shows a different understanding from Ben’s and Manuel’s. He becomes a bit more concrete than Manuel and directly labels the conventional representations of the Dienes-material. His utterance is not necessarily linked to the representations in front, but he highlights special features exclusively connected to the single objects of the Dienes-material (size, form). Frank does not refer at all to the spatial relations of the elements in figure 3.

Until now, a lot has happened during the interaction. Ben starts with a relational interpretation which is limited to the representation of squares. However, Manuel interprets the materials by directly allocating numerical values to the concrete objects. Furthermore, he raises a new focus: the representation with bars. Ben accepts the dissent by expanding his spatial relational interpretation of the squares to the other representation using the bars. Frank also interprets the material and explicitly designates the concrete characteristics of certain symbols. Taking the dispute more closely into consideration, it is maintained by the different utterances. The teacher does not intervene by aiming at a conventional interpretation. Instead, the pupils have an opportunity to discuss a mathematical content. Following the interaction, other pupils join in with their points of views such as “If you add all tens together, then it equals one hundred and twenty. And here those hundreds equal one thousand two hundred (*points at the representation with bars first, then at the other representation*)” and „Um, the top squares are always the hundreds, beneath there are bars, that is the tens, and beneath the ones, so four hundred and seventeen.” Both, the empirically-concrete and the relational interpretations become more and more specific during the interaction.

After John has explained “actually it doesn’t matter, if those are squares or bars. […] You can also use what you haven’t learned yet. Form something from that”, the discussion breaks off. For John, the concrete features do not make any difference. Kevin enhances this point of view by exemplifying this comparison:

K Then you can also, if … I think I know what John means. Then you could also make a poster, where, those four (*points at the four bars at the top*) are dots and then also here (*points at the single bar*) one dot for the tens and, and then here (*points at the seven bars at the bottom*) seven dots. You could also do that actually.

He structurally compares the representations with a not yet produced new representation consisting of dots arranged in the same way. The flexibility of his point of view becomes visible in the confirming statement: "You could also do that, actually," that John formulates subjunctively as a conclusion.

Discussion Episode 1

In this episode, the disputed point does not get solved. Instead, the characteristics of number representation are discussed by using different points of reference, based on distinguishing features on the one hand, and on the relationship between particular elements of the same representation or in comparison to other representations on the other.

Both points of view can be legitimised in a particular manner, are permitted depending on the conditions and are substantiated respectively in the interaction. According to this, by using the Dienes-material the focus is put on the special external characteristics in the first instance. The relations of the Dienes-material are given by combining ten ones to one ten or ten tens to one hundred etc. No matter in which spatial position each symbol is put, the numerical value stays the same. The Dienes-material together with their ideal visual pictures might offer direct relations between the positions of ones, tens and hundreds by counting ten little cubes in a ten bar and ten ten-bars in a hundred square. Then again, as special kind of an additive number system this material is less flexible – it does not emphasise the decimal number *structure*. The system of positions yields advantages; however, most of them are not directly visible, they must be read into the representation. In the position of hundreds, the digit 4 of the number 417 means something completely different than if it was in the position of the tens. This issue is exactly the cause of the mathematical dissent in the course of this episode.

The teacher allows the negotiation of the dissent without breaking it off at once. She intervenes only in very few situations. In this episode, she does not need to, because the pupils' debate arises on its own – with the focus on the mathematical content. Moreover, in this episode the teacher's interventions are directly connected to the mathematical content and targeted at the pupils' utterances, as for instance, in the following situation:

T I think, I know what Frank means. When I turn it like this (*turns the poster at the bottom through 180°*) look, then I can I have a square for each bar. (…) (*repeats pointing four times from the last bar at the bottom* (*in the bar-representation of 417*) *to the last square at the bottom* (*in the square-representation of 417*) (*fig.3*)) Is that, is that what you meant?

The analysis of this short episode shows no teacher-centred interaction between the participants in which the students would have to follow the implicit goals and intentions of the teacher.

4.2 Second Example: Dispute about finding a number in a number-line

In this episode Julian (Ju), Sascha (S), Frank, Janina (Ja) and another teacher (T) than in Episode 1 discuss a number-line with a scale in steps of 20 (produced by Janina in the previous lesson):

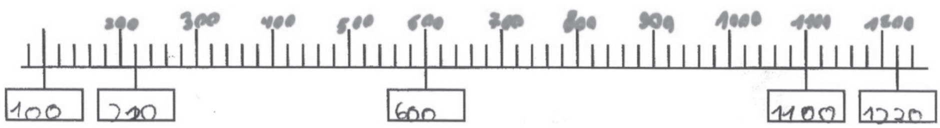

Figure 4 Janina’s number line with inserted numbers

For a better orientation, the pupils write the hundreds at the long bars (Figure 4) before the episode starts.

T What would happen if we looked for the number three-hundred-fifty? Did you write it onto the number-line or look for it?
S It can’t be.
Ju But three-hundred-twenty, thee-hundred and forty, (*is pointing one after the other to the two short bars after the 300-bar*) there in between (*is pointing between the two short bars between the 300- and 400-bar*). There in between.
Ja Yes.

The teacher asks whether or not the number 350 can be found on this number-line. With this question, a problem of the epistemological status of mathematical knowledge is touched upon. From an epistemological point of view (Steinbring, 2005), the teacher introduces a new sign/symbol that needs to be explained by relating this sign to a known explaining reference context. Sascha and Julian interpret the number-line in different ways as explaining reference contexts. Sascha principally argues that numbers can only exist on the number-line when having a pre-given scaling bar to place the number at. In contrast, Julian’s view also allows numbers on the number-line to be placed between scaling bars. In this way, the teacher’s question evolves as a question of debate.

Sascha denies the teacher’s question, at first without any further explanation. In contrast, Julian tries to support his statement by pointing at a location where the number 350 could be exactly and by explaining the conceptual relations of the number-line in a geometrical-relational way. Janina confirms this. In the following interaction, the teacher intervenes by asking Sascha: “Why doesn’t that work?” It is unclear if the disputed point had continued without this intervention. Sascha explains his interpretation more precisely:

114 S Um (…) because (*takes Julian's finger off the number-line and points at the number-line himself*) […] if you um took it in steps of ten because then there would be half that is three-f … three-hundred and fifty but if you had bigger […], because then there (*moves his finger under the number-line from the long bar at the second box to the middle*) would be for example three-hundred (*points at the long bar with the number 300*) and then there four-hundred (*points first at the long bar with the number 400, then at the long bar with the number 500*) at the long one exactly in the middle (..) and because here, um, always ten, twenty, thirty, forty and then fifty (*one after the other, he points at the short bars behind the long bar with the number 300*) then you would already have three-hundred and fifty and with twenties you cannot find it (*1 sec*) twenty forty sixty eighty (*one after the other, he points at the short bars after the long bar with the number 300*).

Sascha does not establish relationships between different number-lines but only refers to the number-line at hand, which he and his group have labelled with numbers (Figure 4), especially at the interval between the label 300 and the label 500. By changing the number 500 into 400 and the steps of twenties into steps of tens hypothetically, he affirms that – if the number-line underlies a decade-structure – the number 350 could be found or marked because a concrete scaling bar exists there to place that number at. He supports this claim in an empirical-descriptive way by referring to the number-line, pointing at it and counting aloud while changing the units. The statement implies a norm which he does not put into words, but it seems to be plausible because of his pointing gestures: You can only find a number if a scaling bar at the number-line exists. This is supported by the use of "always ten" as unit, as he is exemplarily marking the steps of tens up to 50 and by the derivation of the number 350 at the long bar ("at the long one exactly in the middle").

Sascha specifies and rephrases his claim "It can't be" in the last part of his statement by saying "… with twenties you cannot find it". With this, he refers in a reifying way to the actually perceptible objects on the number-line, which he is using as counting-objects. Concerning this statement it remains unclear, if the numerals "twenty forty sixty eighty" refer to a more geometrical interpretation or to counting in steps of twenty, but therefore Sascha indirectly refers to the norm of steps of twenties, which are jointly accepted by the interlocutors, accompanied by the pointing gesture with his finger from bar to bar.

Next, the teacher addresses Julian. Julian reacts to the teacher's demand with several contributions, which are partly interrupted by Sascha:

T Julian, think, I want to find the number three-hundred and fifty on this number-line, can I do that?

Ju I, so when I [count] in steps of ten […] (*turns the number-line to himself*). If it was in steps of ten, then it would be possible but if one [counted] in twenties (*shakes the head*), then it would not be possible.

T So at, um, with these steps (*points underneath the number-line between 300 and 400*) at this number-line (*taps with the hand on the whole number-line*) I cannot find that?
S You can’t (*shakes his head*). No.
J But, in fact, if you [looked] in the middle (*points to the two small bars in the middle of the long bars with the numbers 300 and 400*) but that (*shakes the head*)…

Further, Julian justifies his interpretation of the signs/symbols. At the beginning, he reads the number 350 into the number-line at hand (figure 4). For Julian, the number 350 can be found between the two middle bars between the long bar with the label 300 and the long bar with the label 400. This interpretation can be interpreted in such a way that all numbers can be found on the horizontal line at which the scaling bars are marked. The bars merely help to find them. During the discussion, he changes his view and agrees for a moment with Sascha that the number 350 cannot be found. He confirms the correctness of Sascha’s claim by paraphrasing him. At the end of the scene, Julian tries again to reinforce his claim: “But, in fact, if you [looked] in the middle (*points to the two small bars in the middle of the long bars with the numbers 300 and 400*)”. His geometrical-relational interpretation permits to construct a number on the number-line even if there is no bar for it. But as he cannot explain this interpretation, he rejects it. It seems that his own arguments are neither sufficient nor convincing for himself.

The dispute stops with Julian shaking his head. To sum up the already analysed processes, Sascha’s claim that the number 350 cannot be found in the given number-line and Julian’s counter-argument “in the middle” remain opposed to each other. The question whether or not the number 350 “exists” on the number-line presented (Figure 4), in this case leads to a cooperative argumentation (Klein, 1980). Indeed, Sascha and Julian express their contradiction spontaneously at first. They take their points of view but also try to modulate their different positions to a mutually accepted statement. Looking back at this whole episode, one gets the impression that, with the help of the teacher, Julian is trying to find arguments for or against the teacher’s question if one can find the number 350 on this number-line; so Julian is not only looking for arguments supporting his own position.

Both positions cannot be refuted, but their justification background is different. It remains unclear whether Julian’s argument stays permissible or not, even if it cannot be transferred to be collectively valid, so to speak by the “exclusion principle”. There is no further reflection of Sascha’s statement.

Discussion Episode 2

Also in this second episode, the disputed point is not solved. Instead, some characteristics of number-lines are discussed by using different points of reference in the pupils’ interpretations: Hypothetical conceptual variation in a number-line, the reifying empirical-concrete interpretation which is based on the bars, and the geometrical-relational relations between particular elements of a filled in number-line. All points of view can be legitimised in a particular manner; they are permitted depending on the conditions and they are respectively substantiated in the interaction.

Looking at the teacher's role during the interaction process and how her role refers to the learning of mathematics, it remains unclear if the discussion had continued without the teacher's intervention or if the interlocutors accepted Sascha's utterance as sufficient justification for his claim. The teacher constantly looks at the pupils' interpretations. Furthermore, she takes care of mathematical reasoning in the pupils' interpretation of the presented mathematical signs and symbols. Her interventions initiate a more profound review of the pupils' contributions. The claims, which are at first not justified, become explanations linked to mathematics. Moreover, in the moment in which the number 350 cannot be interpreted in the number-line, the teacher does not take over a transmitting role. The episode shows that a disputed point arising in a mathematical discussion can remain contradicting for students, whereas the teacher, because of her expert knowledge, can relate different interpretations to each other.

5 Summary

In summary, the interpretations show that it is possible to initiate and discuss mathematical crucial points with children at the elementary school age, but it is challenging to focus the attention on special aspects of the interpretation. By giving pupils the possibility of debating about mathematics in the described way, many opportunities for fundamental learning arise, like the exemplary episodes have shown. Both episodes have offered different ways of how students can cope with number representations. The first episode dealt with differences and connections between additive and positional number systems by using and changing the role of known material like the Dienes-material. The second episode dealt with reading numbers into number-lines by varying the unit measure. These ways of working with different sorts of representations offer productive starting points for mathematical debates between teacher and pupils about substantive arithmetical representations and meanings.

With regard to the two research perspectives as expressed in the research questions in the paragraph "data collection" the following first attempts for answers to these research questions can be made: Having a closer look at the teachers' role, they intervene differently during the episodes. Firstly, in episode 1 the teacher *reflects* the importance of the pupil's utterance by asking for a more extensive explanation: "Can you show me that more precisely?" By dealing more profoundly with the statement of the pupil, the teacher's intervention becomes a stimulating effect in the course of the discussion. Secondly, later in the same episode, she passes the interactive utterance back to the students by *rephrasing* Frank's argument. Also in ordinary classroom teaching, in which the teacher has more a role of conveying knowledge, this behaviour is used. The teacher picks up a pupil's contribution and restates it, rephrasing it, formulating it more precisely and introducing more mathematical notions. In this episode, the difference to a classroom setting of conveying mathematical knowledge is the way of dealing with the teacher's contribution: the pupils understand the teacher's intervention as invitation to continue their mathematical negotiation and, by that, to sharpen their understanding and progress their mathematical argumentation skills. The pupils do not try to search in the teacher's reactions for some implicit signals indicating the "correct" knowledge the teacher seems to have in mind.

In the second episode the teacher obtains a different social role. She is coming up with the question: "What would happen if we all would look for the number three-hundred-fifty?" At once, the pupils give two contrary answers, a disputed point evolves. Compared with the first episode, where the disputed point evolves by the pupils' contribution, here the teacher starts the new negotiation. Thus, the teacher *initiates* the discussion. Later in this episode, the teacher intervenes once more. This time in a *provoking* way: "[…] with these steps at this number-line I cannot find that?" Indeed, the teacher provokes once again in the discussion with regard to the supposedly agreed consensus and she reveals an opponent point of view: "No, you can't." and "But, in fact, if you looked in the middle […]" (in this case this is not resumed any more).

All four types of teachers' intervention (*reflecting*, *rephrasing*, *initiating* and *provoking*) can serve as productive catalysts of debate, as far as the participants of the communication understand and use them in the intended way. Thus, the teacher's role is inseparably linked to the manner in which the pupils understand the interaction and how they react themselves. In the presented episodes, the teachers' interventions lead to promote their joint mathematical content-oriented discussion. In this way, such discussions about disputed points might become crucial instances for developing a deeper structural understanding in elementary mathematical topics. It is the goal of the research project *ProFIT* to work out more exactly what and where the crucial points are and the theoretical construct *focusing interaction strategies* to be developed for characterising these crucial points.

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