EXPLORING MATHEMATICS TEACHER KNOWLEDGE FOR TEACHING: MATHEMATICS TEACHERS IN ENGLAND, FRANCE AND GERMANY

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Abstract: *This article explores mathematics teachers' knowledge for teaching, in the Anglo/American, French and German 'scene', and how this may relate to teachers' beliefs and practices as a 'teacher of mathematics'. Using a socio-cultural framework a particular concern was what it meant to be a mathematics teacher in England, France and Germany, and in which ways their knowledge construction was influenced, or shaped, by this. The findings are based on twelve teacher case studies, four in each country, and additional interviewing of 30 teachers, ten in each country. Results show that teacher knowledge for and in teaching was different, in the sense that teachers needed different kinds of knowledge to become effective within their systems and classrooms, and even similar kinds of knowledge (for example, subject knowledge) appeared to be differently perceived in the different educational 'spheres'.*

Key words: *mathematics teachers' knowledge for teaching, teachers' beliefs and practices*

Introduction

In recent years researchers have, again, paid an increasing amount of attention to teacher knowledge, that is teacher knowledge for and in teaching. Professional knowledge of teachers has been investigated from different angles. It is accepted that what teachers know is one of the most important influences on what happens in classrooms. The conceptual tools that teachers posses in order to deal with their work situation depend to a large extend on the cultural and systemic traditions of the educational environment in which they work (Stigler & Hiebert, 1999; Hiebert et al., 2003; Pepin, 1999a). However, there is no consensus on what teachers need to know in order to ensure that student learning is taking place.

In mathematics there is evidence (e.g. Bennett & Turner-Bisset, 1993) that insufficient and poor mathematical knowledge has a negative impact on teaching, and researchers argue about the nature of that knowledge. On the other hand, work at King's College London has found no link between teachers' subject knowledge, measured in terms of academic qualifications, and effective teaching (Askew et al., 1997). Many researchers show that mathematics as knowledge to practice mathematics is distinct from that for teaching mathematics. Ball (2003) argues that mathematics-for-teaching is unlikely to be neither 'more of' or 'to a greater depth than' the knowledge expected of students, but that it is qualitatively different.

> "…knowledge for teaching mathematics is different from the mathematical knowledge needed for other mathematically-intensive occupations and professions. The mathematical problems and challenges of teaching are not the same as those faced by engineers, nurses, physicists, or astronauts. Interpreting someone else's error, representing ideas in multiple forms, developing alternative explanations, choosing a usable definition—these are all examples of the problems that teachers must solve. These are genuine mathematical problems central to the work of teaching." (Ball, 2003, p. 6/7)

Ball and Bass (2003) argue for mathematics-for-teaching to become a distinct branch of mathematics, and there is a growing area of research concerned with this. They argue that mathematics knowledge for teaching is not a watered down version of 'real' (university) mathematics, but a demanding area of mathematical work.

> " …the mathematical knowledge needed for teaching must be usable for ***those*** mathematical problems. Mathematical knowledge for teaching must be serviceable for the mathematical work that teaching entails, from offering clear explanations, to posing good problems to students, to mapping across alternative models, to examining instructional materials with a keen and critical mathematical eye, to modifying or correcting inaccurate or incorrect expositions. The mathematical knowledge needed for teaching, even at the elementary level, is not a watered-down version of 'real' mathematics. Teaching mathematics is a serious and demanding arena of mathematical work." (Ball, 2003, p.7)

From my own research (e.g. Pepin, 1999, 2009; Pepin & Haggarty, 2003), that of colleagues and larger-scale studies such as TIMSS (e.g. Hiebert et al., 2003) it is clear that the work of teaching differs from country to country (e.g. Cogan & Schmidt, 1999). Whilst the quantity and quality of teachers' mathematical knowledge has been an area of great concern (e.g. Ma, 1999), it is, however, less clear how to measure teacher knowledge, what it consists of and how it is comparable across countries. Comparisons of, or simply 'looking into', different knowledges may develop deeper understandings of what we mean by 'knowledge in/for teaching'.

Over more than ten years I have studied mathematics teachers and their curricular practices in mathematics classrooms in different countries, in particular in England, France and Germany. The goal of these studies has been to develop a deeper understanding of what is going on in mathematics classrooms at lower secondary level, especially with respect to teaching and learning mathematics with understanding, and the influence and nature of curricular materials, such as texts used in classrooms. The comparative perspective has helped to highlight particular features of teachers' pedagogic practice, to discover alternatives, and in turn develop a deeper understanding of those features and practices, in addition to stimulate discussion about choices within teachers' immediate environments and countries.

Specific models

In order to set the background for how teacher knowledge may be perceived in different educational 'spheres', I now outline and identify the relations between, some of the most influential models of teacher knowledge in the Anglo/American, the French and the German scene. These will subsequently be used, as a background, to develop a better understanding of the teachers studied in the three environments.

One of the most influential models of teacher knowledge, in particular in the Anglo/American scene, has been provided by Shulman (1986a, b). He asserts that 'where the teacher cognition programme has clearly fallen short is in the elucidation of teachers' cognitive understanding of the subject matter content and the relationships between such understanding and the instruction teachers provide for students'. His interest is mainly in the realm of teachers' subject matter knowledge and the role it plays in teaching, whilst acknowledging that teachers need to possess a 'specialised understanding of the subject matter, one that permits them to foster understanding in most of their students' (Wilson, Shulman, & Richert 1987). Shulman (1987) proposes a framework for analysing teachers' knowledge that distinguishes between different categories of knowledge, and he mainly distinguishes between three kinds of knowledge: subject matter knowledge; pedagogical knowledge; and curricular knowledge. The important part of Shulman's work is the acknowledgement on 'pedagogical content knowledge' which, he claims, helps to fill the gap of the 'missing paradigm'. He describes 'pedagogical content knowledge' as that knowledge 'which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching'.

Shulman's classification of teachers' knowledge has been proven to be very stimulating for research into teachers' cognitions. For example, Bromme (1994) who worked in Germany on mathematics teaching and learning took up Shulman's suggestions, but extended them by two further concepts: the 'philosophy of content knowledge'; and by distinguishing between the knowledge of the academic discipline and that of the subject in school, which includes goals about school and general education. By the 'philosophy of school mathematics' Bromme refers to 'the epistemological foundations of mathematics and mathematics learning and about the relationship between mathematics and other fields of human life', in other words teachers' perceptions on the nature of mathematics and its teaching. Ball et al. (2008) have further developed Shulman's idea of content knowledge for teaching by refining the concept and identifying discernable subdomains such as *specialised content knowledge* which is said to be distinct from *common content knowledge.*

Ernest (1989) explores teachers' knowledge in mathematics teaching, and his model includes teachers' attitudes and beliefs. What is notable in Ernest's model is the importance ascribed to teachers' beliefs, in particular those concerning the nature of the particular subject (mathematics), and concerning the process of teaching and learning the subject. He tries to develop a more fundamental understanding of how mathematics teachers' knowledge, beliefs and attitudes provide a basis for classroom teaching approaches.

Although this model is more detailed than that of Wilson, Shulman and Richert (1987), it shares many of its components with Shulman's model. The comparison provides some measure of support for Ernest's model, since there is evidently a great deal of overlap. Ernest's pedagogical knowledge refers to Shulman's knowledge 'which a teacher uses to transform and represent knowledge of mathematics for teaching' or pedagogical content knowledge (Wilson, Shulman, & Richert, 1987). A key difference between Shulman's and Ernest's models is the apparent neglect of attitudes and beliefs in Shulman's model. However, it seems that beliefs about subject matter are to some extent incorporated into Shulman's 'knowledge of subject matter'.

Brown and McIntyre (1993) developed a model based on the assumption that 'over a period of time experienced teachers have acquired substantial practical knowledge about teaching, largely through their classroom experience rather than their formal training' (p.12). Underlying this approach is the notion that teaching is a craft (rather than a science-based technology) and that experienced teachers have 'craft knowledge' which is accessible to others. They found out that teachers commonly judged their teaching in terms of 'the achievement or maintenance of states of pupil activity which they took to be normally desirable for particular phases and types of lessons (Normal Desirable States-NDS)' and each teacher had their own NDSs for their lessons and for phases of their lessons.

In continental Europe teacher knowledge is part of *Didaktik*, and didactics are often presented in the form of a triangular model (e.g. Houssaye, 1994). Although this model is likely to be limited, it nevertheless allows to establish more clearly the objectives of the study of didactics.

In the French research on the didactics of mathematics there are two distinct, but interrelated, theoretical fields: the theory of *didactical transposition*, based on the work of Chevallard since the 1980s (Chevallard, 1991); and the theory of *didactical situations*, developed by Brousseau (1986) since the beginning of the 1970s and subsequently developed by other researchers. The didactics of mathematics is seen here as the study of the issues of the preparation of mathematics for students.

The two theoretical approaches concern fundamental but different levels of didactical analysis. The concept of the *transposition didactique* (Chevallard, 1991) concentrates on the analysis of the processes that are based on the *savoir savant* (scientific/reference/scholarly knowledge), and how this can be 'transposed' to the *savoir enseigné* (taught knowledge). It is assumed that there exists some identifiable knowledge (*savoir savant*) against which the mathematics taught in schools could be judged or 'legitimised'. Therefore, by adopting the *didactical transposition* approach, one acknowledges the institutions at the source of knowledge.

There has been much criticism of the vagueness of the notion of *savoir savant* (Freudenthal, 1986). Is there a recognised group of professionals (*savants*) who can produce knowledge which is considered 'knowledgeable'? Chevellard (1991) examined relations between the social practice of research in mathematics and that of institutionalised teaching/learning of mathematics in school. In principle, the theory of didactical transposition aims to identify and emphasise the constraints the different actors are subject to, and thus claims to uncover the apparently 'scientific decisions' made (by various decision-makers) as elements of a system following its own rules.

Brousseau's (1986) theory, the theory of didactical situations, is situated at the classroom level. It aims to model teaching situations so that they can be developed and managed in a controlled way. At the basis of this theory is the assumption that 'knowledge exists and makes sense for the cognising subjects only because it represents an optimal solution in a system of constraints' (p. 368). According to Artigue (1994) it is based on a constructivist approach and operates on the principle that knowledge is constructed through adaptation to an environment that appears problematic to the student. Brousseau's theory aims to become a theory for the control of teaching situations in their relationship with the production of mathematical knowledge. The didactic systems are therefore made up of three mutually interacting components: the teacher, the student, the knowledge. The aim is to develop the conceptual and methodological means to control the interacting phenomena and their relation to the construction and functioning of mathematical knowledge in students (e.g. Winslow, 2007).

Like in the French didactics literature, a German core concept in the development of didactics has been the *Didactic triangle* with its three components of the content, the learner and the teacher, in order to structure the field of didactic research and theory. The 'curriculum' tradition is often contrasted to the German didactics (e.g. Kansanen & Pepin, 2005; or Gundem & Hopmann, 1998). Didaktik in Germany has always been a form of philosophical thinking, theorising and the construction of theoretical models (Kansanen & Pepin, 2005), based on 'philosophical traditions of its own with names such as Kant, Herbart, Schleiermacher etc.' (Kansanen, 1995). Kansanen (1995) asserts that

> "*Didaktik* is mainly intended for teacher education and the models are based on a philosophical conception of man and on the nature of research concerning his education. The empirical research results are not a prerequisite for model building, but are used in a corrective way when they are in conflict with the model variables. Research on teaching reflects an empirical tradition and that is why its models are mainly inductive by nature and based directly on research results. Practical conclusions can, of course, be drawn from these models and thus they can also function in teacher education" (Kansanen, 1995, p. 348).

The emergence of research on what constitutes teachers' knowledge in a particular subject has created the *Fachdidaktik* (subject didactics) which denotes the pedagogical transformation of factual content for the purposes of teaching, taking into consideration all factors of the teaching-learning process. The didactics of mathematics became the scientific discipline related to research in mathematics education and the research-related development work (Biehler et al., 1994).

There are at least two conclusions that can be drawn from the comparison of existing representations of knowledge in teaching in the Anglo/American, the French and the German educational scene. Firstly, there appears to be a commonality amongst representations on knowledge in teaching in the sense that it is not seen as static, but as a process of development, that it grows and changes, and that experience in the classroom contributes to its growth and change. Secondly, there seem to be differences in traditions within the research into knowledge of teaching. The German (and French) educational research into teaching appears to be traditionally concerned with philosophical thinking, theorising and the construction of theoretical models, the *Didaktik* (which is nevertheless informed by empirical research). The Anglo/American educational research is to a large extent based on empirical studies, in order to identify and be able to determine factors that are influential for teaching (and learning) and to develop an understanding of the processes involved in teaching and learning (Pepin, 1999c).

Teacher knowledge and 'situatedness' of knowledge

The research literature (e.g. Putnam & Borko, 2000) claims that a 'fundamental shift' in thinking has taken place initiated by the key work of Lave and Wenger (1991). Ideas about the nature of knowledge, and learning, have moved towards what is known as the "situated perspective" (e.g. Greeno, 1997; Lave & Wenger, 1991; Wenger, 1998). Whilst this work largely examined learning in informal settings, it has been influential in mathematics education. Work of Boaler and Greeno (Boaler, 2002; Boaler & Greeno, 2000) show that situated perspectives on learning may offer a different interpretation, representing knowledge "not as an individual attribute, but something that is distributed between people and activities and systems of their environment" (Boaler, 2002, p. 1).

Most of this work has attended to student thinking, and perhaps less to teacher knowledge and learning. Putnam and Borko (2000) have advocated a situated perspective on (teacher) cognition- that is that knowing is situated in physical and social contexts, social in nature, and distributed across persons and tools. They support Ball's argument that the contexts in which students, and teachers, learn and in which we assess what they know are inextricable aspects of their knowledge- thus knowing (and learning) are situated.

> "This professional knowledge is developed in context, stored together with characteristic features of classrooms and activities, organised around the tasks that teachers accomplish in classroom settings, and accessed for use in similar situations." (Putnam & Borko, 2000, p. 13)

Interestingly, using the situated perspective as an analytical tool to view teaching as a distributed activity, Cobb et al. (2003) situated teachers' instructional practices within the institutional settings of the schools (and districts) in which they work. In particular, they emphasise three types of interconnections between various communities of practice within a school or district that involve "boundary encounters, the role of brokers, and the coordination of activity around common boundary objects" (p. 22). The analytic approach that they propose focuses on "teachers' interpretations and understandings while simultaneously treating those interpretations and understandings as situated in and at least partially constituted by the institutional settings in which they work." (p. 13)

Boaler (2002) has taken a situated view that views knowledge as something that is distributed between people and activities and context. Boaler (2000) contends that

> "What is fundamental to the situated perspective is an idea that knowledge is co-produced in settings, and is not the preserve of individual minds. Situated perspectives suggest that when people develop and use knowledge, they do so through their interactions with broader social systems. This may mean that they are learning from a book (written by others) or teacher, or engaging in individual reflection of some socially produced ideas. But the different activities in which learners engage co-produce their knowledge, so that when students learn algorithms through the manipulation of abstract procedures, they do not only learn the algorithms, they learn a particular set of practices and associated beliefs." (Boaler, 2000, p. 3)

Thus, she proposes to shift from a focus only upon knowledge, to one that attends to the inter-relationships of knowledge, practice, and identity. She uses the 'didactic triangle' (not explicitly) with knowledge, identity and practice at its vertices.

Cross-cultural and international studies have tried to explore what and how people know, as a result of learning (Nunes, Schliemann & Carracher, 1993; Stigler & Herbert, 1999; Ma, 1999). These studies highlight the existence of cultural differences, but seemed unable to answer the question of '*how* the cultural shaping of learning takes place' (Sfard & Prusak, 2005, p. 15). Different classroom environments and cultures, constraints and affordances, provided by different settings and opportunities for developing particular mathematical practices, are likely to influence teachers' perceptions of what it means to teach, and learn, mathematics with understanding; and what kinds of knowledges are needed to do that. Teaching mathematics successfully means identifying with and applying the norms of the classroom community which is likely to be different in different contexts, whether they vary from school to school, or from country to country (e.g. Stigler & Hiebert, 1999), and teachers need knowledge of those norms.

The study

In a previous study (Pepin, 1997; Pepin, 1999b, 2002) I have analysed mathematics teachers' work using an ethnographic framework and developed an understanding of the ways teachers worked in their classrooms in England, France and Germany. It emerged that national educational traditions are a large determinant and influence on teachers' pedagogies in the three countries. In a more recent study I (together with Linda Haggarty) investigated mathematics textbooks in the three countries, and connected to that, the ways they were used, by teachers, in English, French and German lower secondary mathematics classrooms (Pepin & Haggarty, 2001; Haggarty & Pepin, 2002). This not only supported some of the earlier findings, but also suggested that the use of curricular materials (such as textbooks), together with the selection of (mathematical) tasks, impacts to a large extent on the mathematical 'diet' offered to students, which in turn is likely to influence students' perception of what mathematics is and what it is to behave mathematically.

For this chapter I have re-analysed some of the data collected over the years in terms of teacher knowledge for and in teaching. The selected data (for this study) consisted of extended lesson observations and interviews with twelve teachers, four in each country, plus shorter observations and interviews with an additional ten teachers in each country. I re-analysed the data on the basis of my understandings of teachers' work, and using a socio-cultural approach to gain new understandings about teacher knowledge in and for teaching.

The main questions asked were:

- What does it mean to be a teacher of mathematics in England, France and Germany?
- In which ways does mathematics teacher knowledge influence/shape the identity of teachers, as teachers of mathematics, in the three countries?

In terms of analysis a procedure involving the analysis of themes similar to that described by Woods (1986) and by Burgess (1984) was adopted. Moreover, at one level I tried to maintain the coherence of the teacher cases through a holistic story of the case that is response validated by participant teachers, and anchored in their own interviews and my observations; at another level , I analysed across teacher cases using my conceptual framework of 'teacher knowledge in/for teaching', testing the hypotheses offered by the different kinds of literature, and building explanations and theorisations grounded in the data; and at a third level looking for similarities and differences of teacher knowledge across country cases. However, due to the additional cross-cultural dimension, it was important to address the potential difficulties with cross-national research, in particular issues related to conceptual equivalence, equivalence of measurement, and linguistic equivalence (Warwick & Osherson, 1973; Pepin, 2002). Particularly important were the findings of Delaney et al. (2008) who compared teacher "mathematical knowledge for teaching" across the US and Ireland, highlighting the value of validity checks of constructs in both contexts. In this respect it was important to locate and understand teacher pedagogic practices and the classroom cultures in England, France and Germany, and it was useful to draw on knowledge gained from earlier research which highlighted the complex nature of teachers' work and classroom environments in the three countries, in addition to potential influences (e.g. systemic developments and educational traditions).

Contextual factors – the system and schooling

In **Germany** mathematics teachers at lower secondary level work in a tri-partite system where 40% of the children of any age group go to the local grammar school (*Gymnasium*) and the remainder is distributed amongst the other school forms, i.e *Realschule*, a technical middle school, and the *Hauptschule*, a secondary modern equivalent. There are also a number of children attending comprehensive schools. Compared with England and France, they represent a low percentage of pupil intake. Anecdotal evidence suggests that the reasons for choosing the Gesamtschule are varied (Nentwig, personal communication). Whilst many pupils find the *Gymnasium* too academically orientated and do not want the practically orientated education of the *Haupt*- or *Realschule*, there are at the same time parents who make a positive decision of sending their children to that school type (because they expect a larger variety of courses and less academic approaches, for example).

Teachers teaching in the different school types have different number of teaching hours, i.e. approximately 25 school hours (each hour is 45 minutes long) for grammar school teachers; and approximately 28 school hours for secondary modern school teachers. The teaching responsibilities are also different for the different school forms. Whereas the grammar school teacher is regarded as the subject specialist teacher for two or three subjects, and is responsible for the teaching of those, the secondary modern teacher is likely to teach many more subjects and has more pastoral care responsibilities. Both are civil servants and paid under different pay spines.

In Germany the class is seen as a unit (albeit within a tri-partite system), rather than promoting a school ethos. Different streams, in different school forms, appear to develop different identities, which is reflected in a different curriculum (for each school type) and in different approaches to the academic and affective in different streams.

In **France** mathematics teachers work in a comprehensive system (*college*, or *lycée*). Depending for which level teachers are educated (*certifié* or *agrégé*), they have different numbers of teaching hours. For example, a 'normal' mathematics teacher (*certifié*) teaches 18 periods per week (where each period has 55 minutes). The main responsibilities for a French mathematics teacher lies with the preparation, teaching and assessment of the mathematics. French teachers are civil servants, and their pay varies with the number of years taught and the assessment by the inspectors which puts teachers on different 'ladders' to go slower or faster up the pay spine.

In France the class is also seen as a unit, rather than the school, and it is perceived that all pupils are entitled to the same curriculum (same textbook for one year group, etc).

In **England** the majority of secondary schools are comprehensive schools, catering either for the 11-16 age range or 11-18. There is also an independent sector with its traditional (selective) prepatory and grammar schools. Mathematics teachers typically work 23 out of 25 hourly periods, and they have many responsibilities beside the teaching of their subject. These can be in terms of pastoral care (of a particular year, for example), or in terms of leadership, i.e. head of the mathematics department.

In contrast to France and Germany English secondary schools try to develop a particular whole school ethos, which is in part influenced by the specialist nature of the school. Teachers feel they have to attend to the needs of the individual child, and setting in mathematics is common practice from year 7 onwards. This means that different mathematics is taught to different groups (sets).

In France one can distinguish between three potential 'routes' as mathematics teachers, and these are linked to teachers' educational background. Firstly, the most esteemed teaching qualification for a French mathematics teacher is the *Agrégation* (concours after Bac +4 or *Maîtrise*), the highest possible degree for teachers of the *collège*, *lycée* and post-secondary institutions. The *professeurs agrégés* are the specialist subject teachers, they are the ***experts (de maths)***, i.e. experts in terms of mathematical knowledge. They have to effect only one year of teacher education, they have the lowest number of teaching hours and rarely any pastoral care commitments. Second, most mathematics teachers in France are *certifiés* (concours after Bac+3 or *Licence*). They are also specialists in their field (and not teaching other subjects), but have to teach more hours than the *agrégés*, in addition to taking on some pastoral care duties. These teachers see themselves as ***profs de maths***. Third, and this is a minority, there are those who work with the teacher education institutions (*Institits Universitaires de Formation de Maitres-*IUFMs) as part-time teacher educators, or with their local authority (*académies*), perhaps on text books, to name but one area. These teachers have been carefully selected by the inspectorate, on the basis of their excellent teaching, and they see themselves as **expert teachers.**

In Germany, the professional routes a mathematics teacher could potentially develop are related to the system of schooling, the tri-partite system, and the associated teacher education. Firstly, the grammar school teacher is the subject specialist (in two subjects) and educated to the highest level in terms of their subject. Perhaps more importantly they are educated at the university (and they are preparing students for university entry) – implying that they have acquired 'Bildung' which is the traditional ideal of Humboldt's 'gebildeter Mensch'- they are the **specialists**. Second, at the *Realschule* teachers are attending to the skill side of mathematics education, in particular technically orientated- they are the **(technical) skills teachers**. Students in the *Hauptschule* mainly prepare for apprenticeships, in addition to those who need a lot of support and pastoral care, for various reasons (e.g. asylum seekers who do not speak German). Thus, and thirdly, these teachers see themselves as **basic skills teachers** and/or ***pastoral carers.*** However, there is also a minority of teachers who work as *Fachleiter* (subject mentors) in schools and in teacher education at the *seminarium*. They are often seen as **expert teachers**, in whichever school form they are working.

In England the potential routes for mathematics teachers are not strictly related to their educational background, in the ways they are in France and Germany. There are those teachers who follow the route of curriculum development and leadership within the mathematics department. This can be done by either taking on particular leading responsibilities within the mathematics department (i.e. head of department), or as Advanced Skills Teacher, a newly developed route that allows salaries to go up to £53'000 per year. These teachers regard themselves as '**skilled (mathematics) teachers'**. The second main route is typically taken by those mathematics teachers who are involved in and want to pursue the direction of '**pastoral carer'**. This means that they may become head of year, or head of House, where they have a number of children 'to look after' in terms of their well being in school. Thirdly, there are those mathematics teachers – **'experienced/expert (mathematics) teachers'** - who work with national or regional organisations (e.g. QCA, QAA, local authority) or as mentors with the universities in teacher education.

Looking across the three countries there are different routes that mathematics teachers can take, and these are to a large extent influenced by the context in which teachers work and study. Interestingly, in France it does not seem to be possible to develop a route as a teacher of mathematics that is not strongly related to the subject matter knowledge. Whether teachers are *agrégé*, *certifié* or expert teachers, also involved in teacher education, the expectations by the system emphasise a heavy reliance on their expertise in terms of mathematical subject knowledge. This is quite different in Germany and England, where teachers can develop and follow a *pastoral carer* route, and the system allows for that in terms of remuneration.

Content knowledge for teaching

When talking about mathematical knowledge for teaching, most people would probably argue that content knowledge matters for teaching. Much research has gone into this, and concepts such as pedagogical content knowledge (Shulman, 1986) have been further developed and refined (e.g. Ball et al., 2008). When asked explicitly about which knowledge is necessary for teaching mathematics, most teachers in the study emphasized mathematical content knowledge (see Fig.1). However, how they 'defined' this was different. English teachers claimed that it means "mathswise to be confident and competent", which includes having sufficient knowledge "if kids go off on a little bit of a tangent which has relevance". They also talked about knowing how to make the mathematics 'digestable' for the pupils/group they teach, to 'adapt any topic in a hundred different ways according to what children (one is) teaching'.

French teachers also pointed to subject knowledge, and interestingly linked it to the ability to 'step back' from the mathematics content. Teachers emphasised the 'distance' (*recul*) that a teacher needs to have, with respect to his/her subject.

> "… to have enough knowledge of the subject, of the mathematics, in order to have enough distance in terms of what one teaches" (Teacher 3, France – my translation)

> "At the pedagogic level one has to accept to step back, in terms of mathematical knowledge …" (Teacher 1, France – my translation)

This is an interesting notion which certainly involves a process of reflection. This reflection is likely to involve consciously thinking about one's experiences with the mathematics, turning ideas over in one's head, looking at things from a different perspective, stepping back to review things, and consciously deciding what one is doing and why. This process is likely to increase knowledge of the subject.

In Germany the *Hauptschule* teachers in the study emphasized the importance of subject knowledge, and elaborated on it in terms of 'conveying the content correctly', whereas *Gymnasium* teachers highlighted aspects of 'logical thinking' in connection with it. Logic was seen as the basis for their mathematics teaching and learning, and teachers worried that pupils often had problems with logic and reasoning. The second most important knowledge aspect was knowledge about the children, in the sense that **all** students must be heard (and not only those with their hands up). In particular the *Hauptschule* teachers stressed the 'background knowledge of the children' in order to 'be able to act educationally sound in problem situations, not only through negative sanctions." (Teacher 1, Germany – my translation)

Thus, it appears that even similar kinds of knowledge, commonly referred to as mathematics 'subject' or 'content' knowledge, are perceived differently in different educational environments. This is most 'visibly' illustrated by the German case teachers who worked within one country and *Land*, but within that *Land* in different school types. This also implies that they have gone through different teacher education. Teacher education for *Hauptschule* teachers shares the patterns of primary school teacher education; it may be argued that it also shares its 'philosophy', that is the 'education of the child' which would explain teachers' discourse and emphasis of pastoral responsibilities in terms of teacher knowledge. *Gymnasium* teacher education focuses on the subject matter (and its teaching), which may explain the emphasis on logic and reasoning in their explanations of subject knowledge.

English teachers	Subject knowledge: Mathematical content knowledge How to make the mathematics 'digestable'/ Adaptation of mathematics according to ind./group
French teachers	Subject knowledge 'Stepping back' from the content (*recul*) Multiplicity of ways of solving a problem and teaching a topic
German teachers	Subject knowledge: HS: to be 'correct' GS: to think logically To know the children: HS: pers. background GS: common problems connected to logic and reas.

Figure 1: Mathematics knowledge in/for teaching

Teachers' beliefs about mathematics and their pedagogic practice

There were three dimensions in terms teachers' beliefs about mathematics, and these underpinned their practices: **importance of conceptual links; process integration; and completeness of pupils' mathematical experiences**.

Teachers in all three countries were concerned about, and concerned with, the coherence of the mathematics taught, in order for a better understanding to be developed. This first dimension was concerned with conceptual links, the inter-connectedness of concepts, and the coherence of mathematical concepts taught. This is supported by the literature, Hiebert and Carpenter (1992) for example, who contend that it is important to be concerned about:

> "…the way information is represented and structured. A mathematical idea or fact is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of the connections." (p. 67).

For French teachers, what was essential for facilitating student understanding involved a number of principles, amongst them that understanding can be characterised by the kinds of relationships or connections that have been constructed between ideas, facts, procedures, for example.

Ma (1999) compared Chinese and US elementary teachers' mathematical knowledge. She found that Chinese elementary teachers perceived mathematical concepts as interconnected, which was in contrast to US colleagues who perceived these concepts as arbitrary collections of facts and rules. She developed a notion of 'profound understanding of fundamental mathematics' (PUFM), an argument for structured, connected and coherent knowledge (Ball et al., 2001), which is 'deep', 'broad' and 'thorough' (Ma, 1999, p. 120) and this was seen as one of the factors for student enhanced mathematical performance. Her PUFM consists of four properties of understanding: basic ideas; connectedness; multiple representations; longitudinal coherence.

> "When it is composed of well-developed, interconnected knowledge packages, mathematical knowledge forms a network solidly supported by the structure of the subject." (Ma, 1999, p. 120)

The second dimension identified concerned a process dimension in teaching mathematics, which was either neglected (as in Germany) or was seen as integral to the learning of the mathematics (as in France). The whole idea about logical thinking was generally also part of that dimension. For example, in France teachers emphasised the process element by preparing cognitive activities for pupils. The idea of 'letting pupils discover' was linked to the teaching of the content, and therefore combining process and content. In England investigations appeared to be done separately, as a separate issue which seemed to be almost like another area of content.

This also resounds with the research literature. Kilpatrick et al. (2001) give a comprehensive view of what they regard as successful mathematics learning. They coin the term "mathematical proficiency" to capture what they think it means for anyone to learn mathematics successfully. Amongst their five strands, *adaptive reasoning* is the 'capacity for logical thought, reflection, explanation, and justification' (Kilpatrick et al., 2001).

Thirdly, teachers were concerned about the coherence of pupils' mathematical experiences. For example, in Germany and in France pupils were expected to reach certain levels at the end of every school year, otherwise they had to repeat the year. On the other hand, in England pupils reached levels of the National Curriculum and some progressed further than others within the same year. This led to a particularity which was not evident in France and Germany, in the sense that English pupils could leave school after year 11 whichever level they had reached. This dimension goes hand-in-hand with the system of schooling and the grouping of pupils within that system, e.g. in the case of England the practice of setting in comprehensive schools. Several studies (e.g. Boaler, 1997) have shown that setting does not enhance achievement, but it means that some students are provided with a different 'mathematical diet' than others, within the same year group.

Teachers' beliefs about mathematics was related to their pedagogic practice, and it may be fruitful to look at 'vignettes' of lessons in each of the three countries, to see how their pedagogic practice relates to knowledge and beliefs. There were certain features in each country that made it characteristic for the teachers.

In previous studies (e.g. Pepin, 1999b), I identified characteristic 'profiles' of classroom situations in England, France and Germany. Teachers assigned significance and value to particular practices which are commonly concerned with pupil engagement and assessment of understanding. For example, in the English classroom, the main aim was to (relatively briefly) explain a particular mathematical notion and let pupils get as much practice as possible. Of particular importance was that pupils were attentive during teacher explanations and subsequently worked on their own whilst teachers attended to individual pupils' needs. The French teachers regarded their main aim as facilitating mathematical thinking by initiating tasks and helping pupils to think around a particular concept, in whole-class conversation, as individuals, or in groups, followed by practice. Thus, of particular importance was that pupils would discover the concept with the help of selected cognitive activities. The main objective in the German mathematics classrooms was to discuss mathematical content. Teachers initiated tasks or discussed exercises from the homework in a conversational style, before giving pupils exercises to practice on their own. They particularly valued that most pupils would be involved in a teacher-led discussion about the mathematical content.

Moreover, there appeared to be particular 'conventions' that all teachers adhered to. For example, teachers in all three countries ask pupils to work on exercises from textbooks for a considerable amount of time, so that pupils can practice what has been explained and teachers can monitor understanding. However, in England, many pupils at Key Stage[2] 4 and almost all at Key Stage 3 had not been issued with a textbook to use in school and at home; they only worked from textbooks during lessons under teacher guidance. Thus, it is likely that the majority of these pupils only ever had access to the textbook in class and consequently had to rely entirely on teacher guided input. In France, the situation was quite different: every pupil had a textbook provided by the school to be used in school and at home. In Germany, pupils had to buy their own textbooks which were selected by schools/teachers from a ministry approved range. Thus, already at the outset there are differences in the roles and importance assigned to textbooks, and for students in terms of access to textbooks.

Looking across the three countries and considering teachers' beliefs about mathematics and their pedagogic practice, it appears that these could be placed on a spectrum, ranging from specialist/professional emphasizing the mathematics (and its teaching and learning) on the one end of the scale, to the pastoral carer prioritizing the well-being of the child on the other end. French teachers' beliefs and practices can then be positioned on the 'mathematics professional/specialist' side of the continuum, considering their involvement with the mathematics, their beliefs in terms of coherence of pupil mathematical experiences and the associated practices. Whereas German *Hauptschule* teachers' beliefs and practices may be placed on the pastoral carer side of it- reflecting their involvement with the 'whole child' and their upbringing. The German grammar school teachers' beliefs and practices may be closely positioned to a French mathematics teachers'. English mathematics teachers' beliefs and practices appeared to be more linked to the skill side of mathematics education and the organization of pupils in their classrooms.

2 In England, compulsory schooling is divided into four key stages. The teachers in this study taught pupils in Key Stage 3 (age 11-14) and Key Stage 4 (14-16).

Conclusions

Considering teacher knowledge and the dimensions that contributed to their knowledge as teachers of mathematics, it is argued that teacher knowledge is 'distributed' across the different dimensions, and relational. This implies that to dissect it into individual components does not reflect the character of it, indeed the very process of dissection may distort it. To give an example: looking across the different teachers' knowledge and their practices (across countries), it cannot be said that teachers knew more, or less, or students learnt less, or more, in any particular classroom. But they taught, and students may have learnt, different mathematics. This means that our understanding of teacher knowledge has to go beyond knowledge, to, for example, their pedagogic practice and the associated beliefs concerning mathematics teaching, and the *relationship between these*. Thus, the site of knowledge production shifts from residing in the teacher to be distributed, in this case to the mathematical practices and the mathematical activities in which they engage students in classrooms.

Looking at teachers' practices across the three countries, it can be argued that teachers tried to establish a personal 'balance', and their practices can be viewed as a personal response to a set of institutional and societal constraints, to a set of teaching traditions –experienced during their time in the system of schooling, and/or in teacher education- and a set of perceptions concerning the teaching and learning of the mathematics.

Furthermore, we contend that different kinds of pedagogic practices, and activities chosen and mediated by teachers, reflect different dispositions (of teachers) towards mathematics, thus shedding light on their relationship with the discipline of mathematics (emerging through the pedagogical practices). These are supported, and perhaps 'produced by', the different educational environments. Looking at the models of teacher knowledge, it could be argued that the German grammar school teacher, the 'pedagogue', can be associated to Ernest's 'Old Humanist', whereas the *prof de maths* in France may be seen as a 'mixture' of Ernest's 'Industrial trainer' and the 'Progressive educator' (Ernest, 1991, p. 138-139).

Moreover, it appears that the three different educational, and cultural, contexts create different contexts for teaching, and learning, of mathematics. It is argued that teacher knowledge in and for teaching was different in the English, French and German classrooms studied, in the sense that teachers needed different kinds of knowledge within their respective environments. Even similar kinds of knowledge (for example, subject knowledge) appeared to be differently situated in the different culturally figured environments. Whereas in one context (England) content knowledge was seen to serve the adaptation of the mathematics to become 'digestable' for the students- a practical consideration, in another (France) the emphasis was on the development of the knowledge residing within the teacher, and reflection (stepping back) was necessary for that. The German cases also illustrated the influence of context and environment on the knowledges perceived to be appropriate for teaching mathematics: in one context (HS) subject knowledge was about the 'correctness' of the mathematics; in another (GS) about 'thinking logically' – different natures of subject knowledge.

It may be interesting to consider to what extent the context in which the teachers work, perhaps the systemic features of the three countries here, may allow mathematics teachers to shift and develop different 'knowledges', associated with their developing beliefs and practices, without running the risk of becoming less effective. Under which conditions would teachers be able to develop new 'knowledges', as teachers of mathematics? What kinds of knowledge are needed to redefine oneself, to enable, or scaffold, a transformation process? If change is desired, these seem to be pertinent questions for every policy maker to consider.

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